Analytical Derivation of Complex Cell Properties in V1 from the Slowness Principle

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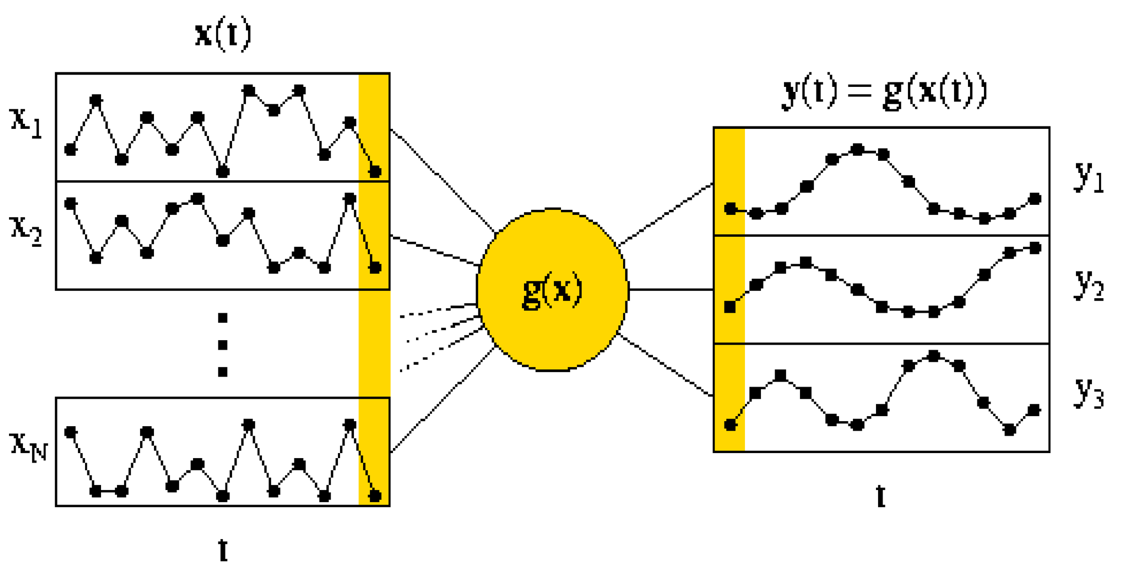
Abstract

Cells in the primary visual cortex (V1) are perceived as edge or line detectors and classified as simple or complex cells depending on their sensitivity to the spatial position of their prefered input pattern. A number of theoretical models put forth the proposition that their receptive fields form in a self-organized manner in order to optimize certain computational objectives. Several groups (Földiak (Neural Computation 3:194), Kayser et. al. (Neural Computation 15:1751), Berkes and Wiskott (Journal of Vision 5:579)) have reported simulations of complex cells based on the principle of temporal slowness. Here, we present an analytical model for the simulations of Berkes and Wiskott. The model reproduces the structure of the optimal stimuli and the orientation and frequency selectivity of the simulated cells. Interestingly, the results depend solely on the transformations and not the content of the image sequences used for training.

Slow Feature Analysis



Learning Task:

Given a function space F and a vectorial input signal $\boldsymbol{x}(t)$, find a set of instantaneous input-output functions $g_j(\boldsymbol{x}) \in F$, whose output signals $y_j(t) = g_j(\boldsymbol{x}(t))$ minimize

$$\Delta(y_j) = \langle \dot{y}_j(t)^2 \rangle_t$$

under the constraints

$$\langle y_j(t) \rangle_t = 0 \qquad (\textit{zero mean})$$

$$\langle y_j(t)^2 \rangle_t = 1 \qquad (\textit{unit variance})$$

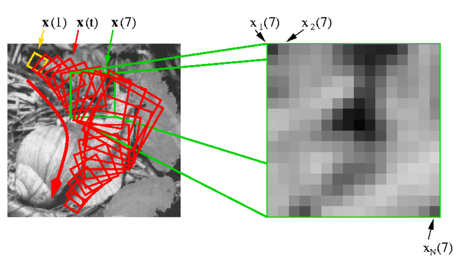
$$\forall i < j : \langle y_i(t) y_j(t) \rangle_t = 0 \qquad (\textit{decorrelation})$$

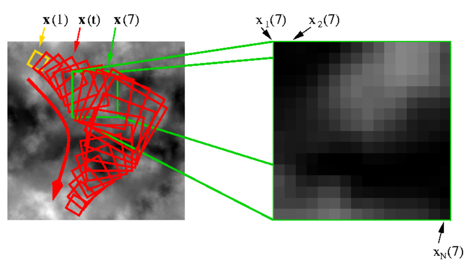
Wiskott & Sejnowski (Neural Computation 14:715)

Computer Experiment

Input Signal: Image sequences generated from natural images/colored noise images by shifting, rotating and zooming a quadratic frame.

Natural Images → Image sequences ← Colored Noise Images





Function space: Polynomials of degree 2 $\quad g_j(\boldsymbol{x}) = \boldsymbol{c}_j + \boldsymbol{f}_j \cdot \boldsymbol{x} + \frac{1}{2} \boldsymbol{x}^T \boldsymbol{H}_j \boldsymbol{x}$

Observation: Structure of the solution is

- largely independent of the structure of the images.
- determined by the transformations.

Berkes & Wiskott (Journal of Vision 5:579)

Analytical Model

We employ Lie group theory for modeling the transformations used to generate the input data. The basic assumptions of our model are

- Continuous images $x(\boldsymbol{r})$
- Quadratic functions

$$g_j(x(\boldsymbol{r})) = \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} g_j(\boldsymbol{r}, \boldsymbol{r}') x(\boldsymbol{r}) x(\boldsymbol{r}') \, d\boldsymbol{r} \, d\boldsymbol{r}'$$

- Invariance of the input data statistics with respect to translation, rotation and zoom
- Decorrelation of different transformations

By means of variational calculus, we can prove that the optimal functions are the solutions of an operator eigenvalue problem:

$$\hat{\boldsymbol{D}} g_j(\boldsymbol{r}, \boldsymbol{r}') = \Delta_j g_j(\boldsymbol{r}, \boldsymbol{r}')$$

where $\Delta_j = \Delta(y_j)$ and

$$\hat{\boldsymbol{D}} = -\langle v^2 \rangle (\nabla_r + \nabla_{r'})^2 - \langle \omega^2 \rangle (\boldsymbol{r} \times \nabla_r + \boldsymbol{r}' \times \nabla_{r'})^2 - \langle \alpha^2 \rangle (\nabla_r \cdot \boldsymbol{r} + \nabla_{r'} \cdot \boldsymbol{r}')^2$$

In the simplified case of translation-invariant functions $g_j(\boldsymbol{r}, \boldsymbol{r}') = g_j(\boldsymbol{r} - \boldsymbol{r}')$ this problem can be solved analytically.

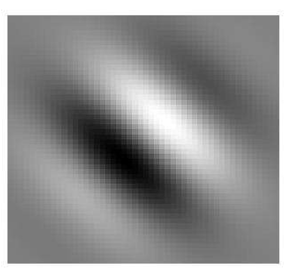
Receptive Field Properties

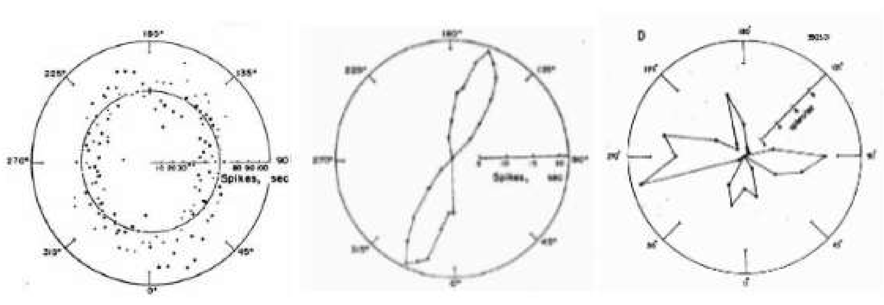
Interpreting the optimal functions as nonlinear receptive fields, we can analyze their behaviour in terms of optimal stimuli and orientation and frequency selectivity and compare the properties of the analytical solutions with those of simulations and physiological experiments:

Optimal Stimuli | Orientation Selectivity | Frequency Selectivity

Physiology

edges
gratings
Gabor wavelets

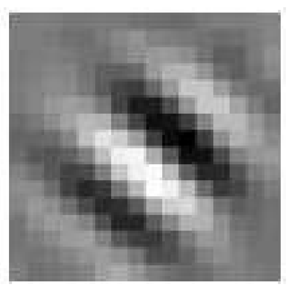


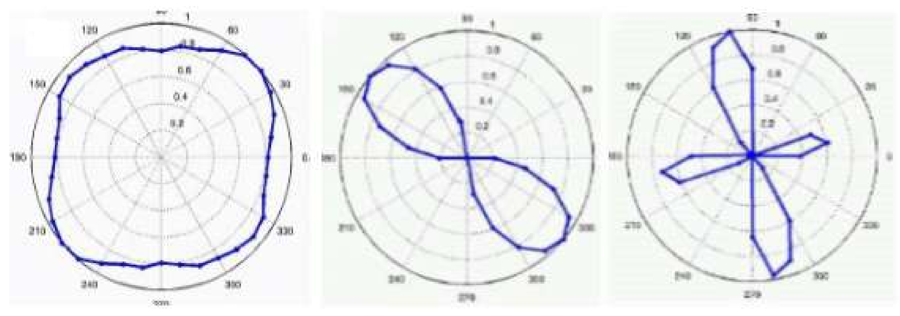


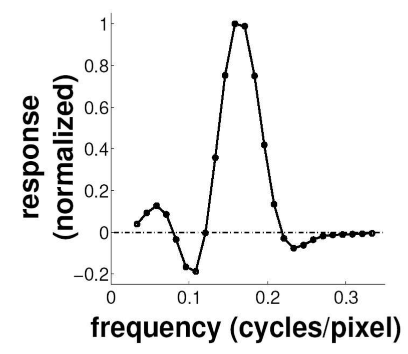
DeValois et. al. (Vision Research 22:531)

Computer Experiments

similar to
Gabor wavelets







Analytical Model

plane waves