Slowness: An Objective for Spike-Timing-Dependent Plasticity?

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Abstract

Ever since the experimental discovery of the peculiar spike timing dependence of Hebbian synaptic plasticity, a lot of research has been dedicated to the question of its functional interpretation. Here, we investigate the learning dynamics of STDP for a linear Poisson neuron and show that it is not the learning window alone that is functionally relevant, but rather its convolution with the post-synaptic potential. This offers the interpretation of the asymmetric shape of the learning window as a deconvolution mechanism for neuronal low-pass filtering. We then show that the learning window can be split into two functionally separate components that are sensitive to the reversible and the irreversible aspects of the input statistics, respectively. Under certain conditions the symmetric component of the learning window can be interpreted as a mechanism for learning slowly varying features of the input data. This is in line with several abstract learning rules that were previously proposed as mechanisms for learning invariant sensory representations.

Spike-timing-dependent plasticity in linear Poisson neurons

Inhomogeneous Poisson input spike trains: $\langle S_i^{\text{in}} \rangle = r_i^{\text{in}}(t) = \bar{r}_i + x_i(t)$

Linear Poisson neuron:

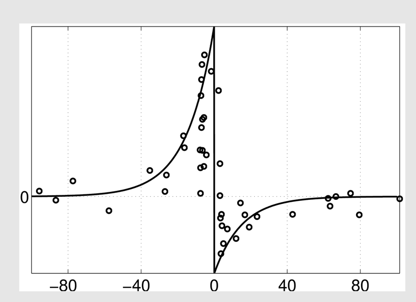
Output Firing rate: $v(t) = \sum_i w_i [\epsilon \circ S_i^{\text{in}}](t)$

Average firing rate: $r^{\text{out}}(t) = \langle v(t) \rangle = \sum_i w_i [\epsilon \circ r_i^{\text{in}}](t)$

Model for STDP:

$$\Delta w_i = \int W(\tau) S_i^{\text{in}}(t+\tau) S^{\text{out}}(t)\, \mathrm{d}t\, \mathrm{d}t'$$

- Interaction between all spike pairs
- Weight modification independent of the weight



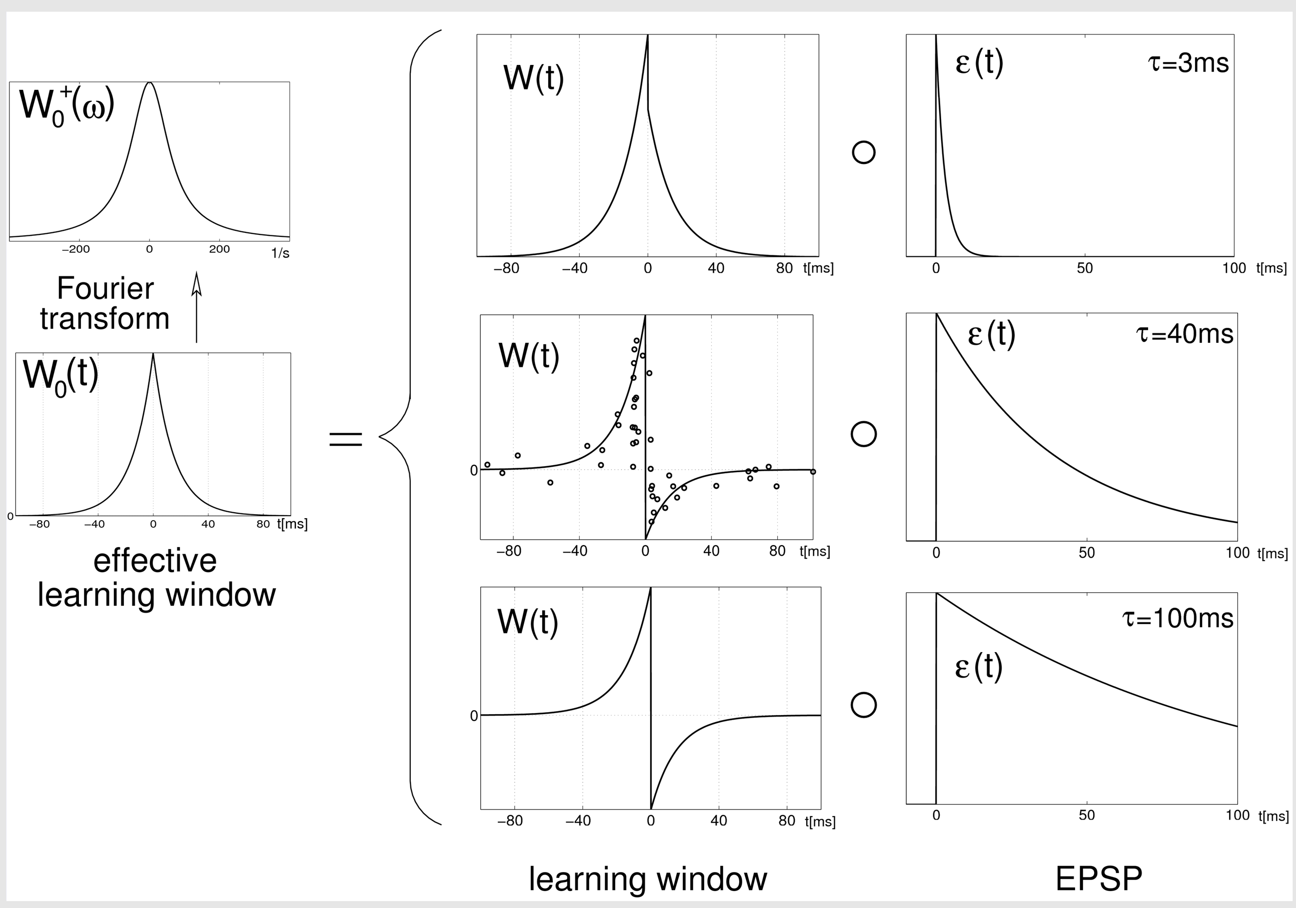
Data from Bi & Poo (1998)

Ensemble averaged weight dynamics:

$$\Delta w_i = \sum_j \left[\int [W \circ \epsilon](\tau) C_{ij}(\tau) \mathrm{d}\tau \right] w_j + F(\{\bar{r}_i\}) = \sum_j A_{ij} w_j + F(\{\bar{r}_i\})$$

with the cross-correlation functions $C_{ij}(\tau) = \langle x_i(t+\tau) x_j(t) \rangle_t$

Temporally asymmetric learning as a deconvolution mechanism



Depending on the EPSP, different learning windows can implement the same learning rule

Data taken from Bi & Poo (1998)

Functional components of the learning window for different input statistics

The correlation functions can be split into a reversible and an irreversible component:

$$C_{ij}(\tau) = C_{ij}^+(\tau) + C_{ij}^-(\tau) \quad \text{with} \quad C_{ij}^{\pm}(\tau) = \pm C_{ij}(-\tau)$$

For symmetry reasons, the weight dynamics become

$$\Delta w_i = \sum_j \Big[\underbrace{\int W_0^+(\tau) C_{ij}^+(\tau) \mathrm{d}\tau}_{A_{ij}^+} + \underbrace{\int W_0^-(\tau) C_{ij}^-(\tau) \mathrm{d}\tau}_{A_{ij}^-} \Big] w_j$$

with a similar decomposition of the learning window $W_{ij}^{\pm}(\tau) = \pm W_{ij}(-\tau)$.

→ The symmetric/antisymmetric component of the learning window target reversible/irreversible aspects of the input statistics.

Because $C_{ij}(\tau) = C_{ji}(-\tau) \Rightarrow A_{ij}^{\pm} = \pm A_{ji}^{\pm} \Rightarrow \sum A_{ij}^+ w_j \perp \vec{w}$

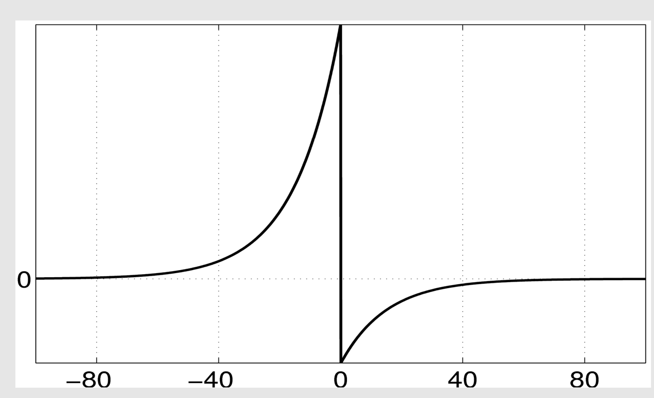
→ The antisymmetric component of the learning window does not favor stable weight dynamics → oscillatory dynamics?

The symmetric component: A means for learning slow signals?

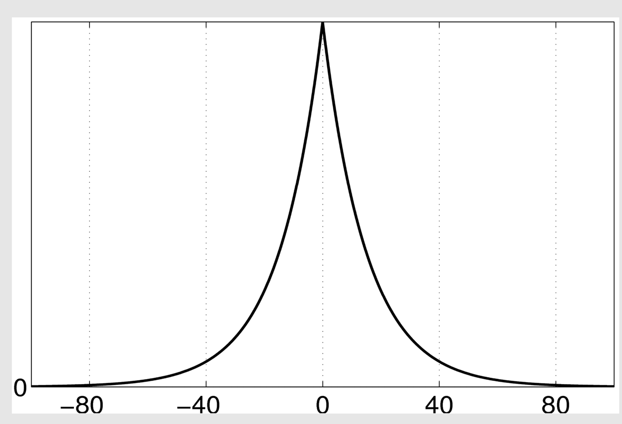
For reversible learning statistics or a symmetric (effective) learning window, the learning dynamics can be interpreted as a gradient ascent on the objective function

$$\Psi(w) = \frac{1}{2} \sum_{ij} w_i A_{ij}^+ w_j = \frac{1}{2} \int W_0^+(\tau) \langle y(t) y(t+\tau) \rangle_t \mathrm{d}\tau \quad \text{with} \quad y(t) = \sum_i w_i x_i(t)$$

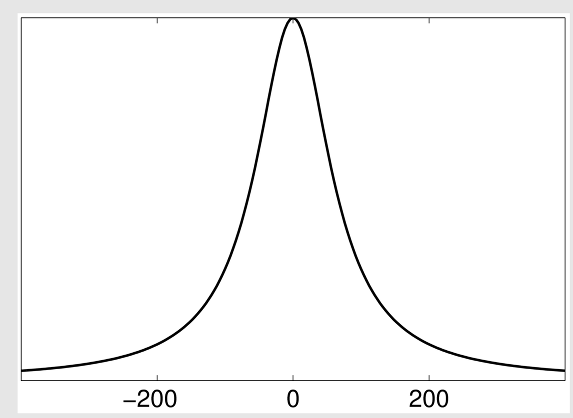
$$= \frac{1}{2} \int \tilde{W}_0^+(\omega) |\tilde{y}(\omega)|^2 \mathrm{d}\omega$$



Effective STDP learning window W_0



Symmetric component W_0^+ of the effective learning window



Fourier transform of W_0^+
→ Effective filter in the objective function

If the symmetric component of the effective learning window is a low-pass filter, the learning rule prefers slow signals.

Summary

We analyzed a model for STDP for a linear Poisson neuron and showed that

- The learning dynamics are not governed by the STDP learning window alone but rather by the convolution of the learning window with the EPSP. This offers the interpretation of the antisymmetry of the learning window as a means for deconvolving neuronal low-pass filtering.
- The effective learning window can be split into a temporally symmetric and an antisymmetric component. These components are sensitive to reversible and irreversible input statistics, respectively.
- For reversible input statistics, STDP can under certain conditions be interpreted as a mechanism for learning slowly varying components of the input signals.