Humboldt-Universität Berlin
Fachinstitut für Theoretische Biologie

Computer Praktikum 5
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Computational Neuroscience II: Foundations of Neural Coding

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! This is the last assignment of the Programming Course. The projects will be available at the beginning of June at http://itb.biologie.hu-berlin.de/~kempter/Teaching/2003_SS/. We will give a brief introduction to all projects on June 5, 2003 during the regular course hours. You can sign up for one project, and start working on it right away.

12. **Random distributions**

The random number generator of Matlab, >> `rand`, produces uniformly distributed random numbers (called u here) in the open interval $]0,1[$. Try! Then restart Matlab and try again. Do you get the same 'random' numbers?

How can we obtain random variables t that are drawn from any given probability distribution $f(t)$? This can be done by solving the integral equation

$$\int_{-\infty}^{t} \mathrm{d}t' f(t') = u, \tag{1}$$

for t, where $u =$ `rand` is a given random number. If you are ambitious to understand this, proceed with exercise 15.

13. **Homogeneous Poisson Process**

- Derive an algorithm for producing random numbers t drawn from the probability distribution $f(t) = \lambda_0 \exp(-\lambda_0 t)$ for $t \geq 0$ and $f(t) = 0$ for $t < 0$ (solution: $t = -[\log(1-u)]/\lambda_0$). The random numbers t can be interpreted as the intervals between events (or spikes) of a *homogeneous Poisson process* with rate λ_0, and $f(t)$ is the inter-spike interval (ISI) distribution.
- Implement a Matlab function named `Poisson` that takes the rate λ_0 as input, and returns an ISI.
- Produce a 10-second Poisson spike train with rate $\lambda_0 = 10$ Hz using a `while` loop that terminates when the 10-second period is over (hint: `>> help while`).
- Plot the histograms of the above spike train (bin width 50 ms) and ISI distribution (bin width 2 ms).
- Produce a 10-second Poisson spike train with rate 10 Hz and absolute refractoriness of (unphysiological) 20 ms. Again, plot the histogram and ISI distribution.

14. **Inhomogeneous Poisson Process**

We start from the inter-spike interval distribution $f(t) = \lambda_0 \exp(-\lambda_0 t)$ of a homogeneous Poisson process that has generated a set $\{s_i\}$ of spike times $(1 \leq i \leq N)$ with a constant rate λ_0 (see exercise 13). An *inhomogeneous Poisson process* produces spikes with a time-dependent rate $\lambda(t)$ that can be any smooth enough function. The set $\{t_i\}$ of spike times of the inhomogeneous Poisson process can be obtained from the homogeneous Poisson process $\{s_i\}$ by using the integral transformation $s_i = \int_{t_0}^{t_i} \mathrm{d}t\, \lambda(t)/\lambda_0$, or the recursion formula $s_i = s_{i-1} + \int_{t_{i-1}}^{t_i} \mathrm{d}t\, \lambda(t)/\lambda_0$. In order to generate and analyze an inhomogeneous Poisson process, please proceed as follows:

- Produce a 1-second homogeneous Poisson spike train $\{s_i\}$ with rate $\lambda_0 = 100$ Hz.
- Write the Matlab function `timewarp` that transforms the set of spike times $\{s_i\}$ into the set $\{t_i\}$ of spike times of an inhomogeneous Poisson process with rate $\lambda(t) = r\,[1 + \cos(2\pi\, f\, t)]$ where $r = 100$ Hz and $f = 3.3$ Hz. *Hint: Numerically integrate the above transformation with step width $\Delta t = 2$ ms by means of a `while` loop that terminates as soon as the sum $I(n_i) = \sum_{n=0}^{n_i} \Delta t\, \lambda(t_0 + n\Delta t)/\lambda_0$ is larger than s_i. At this point we have $t_i \approx n_i \Delta t$.).*
- Produce 20 more 1-second spike trains $\{t_i\}$ at rate $r = 10$ Hz and plot the average rate $\langle \rho(t) \rangle$.
- Calculate and plot the average spike autocorrelation function (see last week's Solution) of the 20 spike trains. What do large peaks near ± 1 s mean?
- Calculate and plot the average spike stimulus correlation function

$$Q_{\rho\lambda}(s) = T^{-1} \int_0^T \mathrm{d}t\, \langle \rho(t+s)\, \lambda(t) \rangle$$

  Hint: T=1 s.

15. **One for the specialists** (you won't need Matlab)

- Invert the integral Equation (1) for two more examples:
  a) $f(t) = 1/\Delta$ for $0 \le t \le \Delta$ and $f(t) = 0$ otherwise.
  b) $f(t) = t/\Delta$ for $0 \le t \le \sqrt{2\Delta}$ and $f(t) = 0$ otherwise.
- Plot both the probability density $f(t)$ and the cumulative distribution function $\int_{-\infty}^{t} \mathrm{d}t'\, f(t')$ for one of the two examples above ($-1 \le t \le 3$, $\Delta = 2$), and provide a graphical explanation for the solution of Equation (1).
- Try to derive Equation (1). Start with the normalization condition $\int f(t')\mathrm{d}t' = 1$ and proceed with the appropriate integral parameter transformations.

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Processing and discussion of the exercises on 22.05.2003, $8^{15} - 10^{00}$ am.