Computational Neuroscience II: Foundations of Neural Coding
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This is the last assignment of the Programming Course. The projects will be available at the beginning of June at http://itb.biologie.hu-berlin.de/~kempter/Teaching/2003_SS/. We will give a brief introduction to all projects on June 5, 2003 during the regular course hours. You can sign up for one project, and start working on it right away.

12. Random distributions
The random number generator of Matlab, >>> rand, produces uniformly distributed random numbers (called \( u \) here) in the open interval \([0, 1[\). Try! Then restart Matlab and try again. Do you get the same ‘random’ numbers?

How can we obtain random variables \( t \) that are drawn from any given probability distribution \( f(t) \)? This can be done by solving the integral equation

$$\int_{-\infty}^{t} dt' f(t') = u,$$

for \( t \), where \( u = \text{rand} \) is a given random number. If you are ambitious to understand this, proceed with exercise 15.

13. Homogeneous Poisson Process

- Derive an algorithm for producing random numbers \( t \) drawn from the probability distribution \( f(t) = \lambda_0 \exp(-\lambda_0 t) \) for \( t \geq 0 \) and \( f(t) = 0 \) for \( t < 0 \) (solution: \( t = -[\log(1 - u)]/\lambda_0 \)). The random numbers \( t \) can be interpreted as the intervals between events (or spikes) of a homogeneous Poisson process with rate \( \lambda_0 \), and \( f(t) \) is the inter-spike interval (ISI) distribution.
- Implement a Matlab function named \texttt{Poisson} that takes the rate \( \lambda_0 \) as input, and returns an ISI.
- Produce a 10-second Poisson spike train with rate \( \lambda_0 = 10 \) Hz using a \texttt{while} loop that terminates when the 10-second period is over (hint: >>> \texttt{help while}).
- Plot the histograms of the above spike train (bin width 50 ms) and ISI distribution (bin width 2 ms).
- Produce a 10-second Poisson spike train with rate 10 Hz and absolute refractoriness of (unphysiological) 20 ms. Again, plot the histogram and ISI distribution.

14. Inhomogeneous Poisson Process

We start from the inter-spike interval distribution \( f(t) = \lambda_0 \exp(-\lambda_0 t) \) of a homogeneous Poisson process that has generated a set \( \{s_i\} \) of spike times \( (1 \leq i \leq N) \) with a constant
rate \lambda_0 \) (see exercise 13). An \textit{inhomogeneous Poisson process} produces spikes with a time-dependent rate \( \lambda(t) \) that can be any smooth enough function. The set \( \{t_i\} \) of spike times of the inhomogeneous Poisson process can be obtained from the homogeneous Poisson process \( \{s_i\} \) by using the integral transformation \( s_i = \int_{t_0}^{t_i} dt \lambda(t)/\lambda_0 \), or the recursion formula \( s_i = s_{i-1} + \int_{t_{i-1}}^{t_i} dt \lambda(t)/\lambda_0 \). In order to generate and analyze an inhomogeneous Poisson process, please proceed as follows:

- Produce a 1-second homogeneous Poisson spike train \( \{s_i\} \) with rate \( \lambda_0 = 100 \text{ Hz} \).
- Write the Matlab function \texttt{timewarp} that transforms the set of spike times \( \{s_i\} \) into the set \( \{t_i\} \) of spike times of an inhomogeneous Poisson process with rate \( \lambda(t) = r [1 + \cos(2\pi f t)] \) where \( r = 100 \text{ Hz} \) and \( f = 3.3 \text{ Hz} \). \textit{Hint: Numerically integrate the above transformation with step width \( \Delta t = 2 \text{ ms} \) by means of a while loop that terminates as soon as the sum \( I(n_i) = \sum_{n=0}^{n_i} \Delta t \lambda(t_0 + n\Delta t)/\lambda_0 \) is larger than \( s_i \). At this point we have \( t_i \approx n_i\Delta t \).}
- Produce 20 more 1-second spike trains \( \{t_i\} \) at rate \( r = 10 \text{ Hz} \) and plot the average rate \( \langle \rho(t) \rangle \).
- Calculate and plot the average spike autocorrelation function (see last week’s Solution) of the 20 spike trains. What do large peaks near \( \pm 1 \text{ s} \) mean?
- Calculate and plot the average spike stimulus correlation function

\[
Q_{\rho\lambda}(s) = T^{-1} \int_0^T dt \langle \rho(t+s) \lambda(t) \rangle
\]

\textit{Hint: } \( T = 1 \text{ s} \).

15. **One for the specialists** (you won’t need Matlab)

- Invert the integral Equation (1) for two more examples:
  a) \( f(t) = 1/\Delta \) for \( 0 \leq t \leq \Delta \) and \( f(t) = 0 \) otherwise.
  b) \( f(t) = t/\Delta \) for \( 0 \leq t \leq \sqrt{2\Delta} \) and \( f(t) = 0 \) otherwise.
- Plot both the probability density \( f(t) \) and the cumulative distribution function \( \int_{-\infty}^t dt' f(t') \) for one of the two examples above \( (-1 \leq t \leq 3, \ \Delta = 2) \), and provide a graphical explanation for the solution of Equation (1).
- Try to derive Equation (1). Start with the normalization condition \( \int f(t') dt' = 1 \) and proceed with the appropriate integral parameter transformations.

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