Humboldt-Universität zu Berlin
Institute for Theoretical Biology

Exercise Set 7
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Computational Neuroscience IV: Analysis of Neural Systems

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Correlations of Multiple Spike Trains

The effort to understand how populations of neurons represent sensory perception or motor action has challenged neuroscientists to develop techniques to record from many neurons simultaneously. At the most basic level, one hopes to discern the functional connectivity of neurons in the network by such multiple recordings, telling apart those neurons that are wired together via synapses connected from those that are not.

Spike trains are often binned in time and then represented by binary vectors: a '1' indicates a spike in a time bin Δt, a '0' indicates no spike. A precise formulation requires representing a spike by the value Δt^{-1} instead of simply a '1' and the absence of a spike by the value of zero. The raw cross-correlogram of two spike trains on the rth trial of an experiment is then given by

$$C^r_{\Delta t}(\tau) = \sum_{t=-\infty}^{\infty} S^r_1(t) S^r_2(t+\tau)\Delta t \doteq [S^r_1 * S^r_2](\tau) \tag{1}$$

where $S^r_1(t)$ and $S^r_2(t)$ are the spike trains of the first and second cell in the rth trial at time t. We take the symbol $*$ as a shorthand for computing the correlation. In the limit of infinitely small time bins, we replace the sum by an integral

$$C^r(\tau) = \int_{-\infty}^{\infty} \rho^r_1(t)\, \rho^r_2(t+\tau)\, \mathrm{d}t. \tag{2}$$

In this limit, the binary time series $S(t)$ is replaced by a sum over delta distributions $\rho(t) = \sum_i \delta(t - t_i)$ where t_i are the spike times.

If two cells react to the same stimulus, repeated trial after trial, the spike trains will naturally covary in time. The average of a spike train of cell i over N trials is defined as $\langle S^r_i(t)\rangle = N^{-1} \sum_{r=1}^{N} S^r_i(t)$, which is an estimate for the average instantaneous firing rate. To obtain the normalized, trial-averaged cross-correlogram $V_{\Delta t}(\tau)$ (also known as the covariogram) we subtract the *average* response $\langle S^r_i(t)\rangle$ from $S^r_i(t)$ for each spike train before computing the correlogram in equation 1, and then perform an average over all trials,

$$V_{\Delta t}(\tau) \doteq \langle [(S^r_1 - \langle S^r_1\rangle) * (S^r_2 - \langle S^r_2\rangle)]\,(\tau)\rangle$$

1. Show that

$$\sum_{\tau=-\infty}^{\infty} C^r_{\Delta t}(\tau)\,\Delta t = n^r_1 n^r_2 \qquad \text{and} \qquad \int_{-\infty}^{\infty} C^r(\tau)\,\mathrm{d}\tau = n^r_1 n^r_2,$$

where n^r_i is the total number of spikes fired by cell i during trial r.

2. Show that the covariogram V can be written as

$$V_{\Delta t}(\tau) = \langle C^r_{\Delta t}(\tau)\rangle - K(\tau)$$

with $K(\tau) = [\langle S^r_1\rangle * \langle S^r_2\rangle](\tau)$. K is sometimes called the *shuffle corrector* for the raw cross-correlogram.

3. Without any stimulus, suppose that two cells fire randomly and independently at a background rate B of 20 Hz. The presentation of a stimulus leads to an average of 20 additional spikes per second in each of these cells, such that whenever the first cell fires an additional spike, the second cell also fires a spike within a Gaussian time window of standard deviation $\sigma = 2.5$ ms. We thus observe stimulus-induced spike synchrony, which is not perfect in the sense of exact coincidence of spikes in time, but is completely reliable in terms of spike-to-spike correspondence between the two cells, regardless of the instantaneous firing rate. The instantaneous firing rate corresponds the probability at time t of seeing a spike (a '1' in the binary vector of the spike train) in the time bin centered at t, normalized by dividing by the bin width. In the limit of infinitely short time bins, we will never observe more than one spike in a time bin. Suppose that the firing rate of *both* cells can be described (in the limit of bin width $\Delta t \to 0$ and number of trials $N \to \infty$) by the following mathematical model

$$f(t) = \underbrace{\mathrm{R}(t)}_{\text{Stimulus induced}} + \underbrace{\mathrm{B}}_{\text{Background}}$$

on each and every trial. Background firing B and stimulus-induced firing $R(t)$ are assumed to be independent. Write down a mathematical expression for the covariogram, starting from an integral formulation such as in eq. 2 instead of a sum over time bins, and argue why the covariogram should be independent of the exact form of $\mathrm{R}(t)$.

4. There are many processes that modulate the firing rates of whole populations of neurons, such as vigilance, attention, or even reward signals (conveyed via dopaminergic signals). These processes lead to slow co-modulations of the firing rates of multiple cells. On the other hand, many stimuli, such as visual flicker stimuli, lead to very transient responses in cortex. Thus, for instance, during a single trial a stimulus can cross the 'receptive field' of a neuron many times, each time giving rise to a short-lived response.

Supposing that we can describe the slow variations by scalar gain factors, we could express the firing rate on a trial-by-trial basis by

$$\underbrace{f^r(t)}_{\text{firing rate during trial } r} = \underbrace{\xi^r \mathrm{R}(t)}_{\text{Stimulus induced}} + \underbrace{\beta^r \mathrm{B}}_{\text{Background}}$$

where ξ^r and β^r are the gain factors for the rth trial. Imagine, for instance, that on trials in which the subject fully attended the trial task, the overall response in both cells is increased by some factor, without changing the shape of the response as a function of time, whereas the response drops in unattended trials.

For simplicity's sake, let us ignore the background firing rate by setting $\mathrm{B} = 0$. Furthermore, assume that there is *no* intrinsic synchrony between cells in their firing patterns, but that the firing rates of the cells $R(t)$ are identical. Compute the covariogram $V(\tau)$ for

$$R(t) = t \exp(-t/\lambda) \qquad \text{for } t \geq 0$$

and $\lambda = 5$ ms, expressing the result in terms of the covariance of the gain factors $\mathrm{cov}(\xi_1^r, \xi_2^r) \doteq \langle \xi_1^r \xi_2^r \rangle - \langle \xi_1^r \rangle \langle \xi_2^r \rangle$. (Advice: use integrals, instead of sums over time bins. You will need to compute the correlation integral of $R(t)$ with itself.) Sketch the result for a positive covariance of one and compare to the result you got in problem **3**.

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Problems handed out on Monday, 19.06.2006. Solutions to be handed in by Monday, 26.06.2006.
Discussion and presentation of the problems on Friday, 30.06.2006, $8^{30} - 10^{00}$ am in front of room 2317 I-W (Zwischengeschoß).